TRANSMISSION OF SOUND THROUGH A FINE GRID

(ZVUKOPROVODNOST' CHASTOI RESHETKI)

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M.I.GUREVICH

(Moscow)

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A formula due to G.D. Maliuzhints for calculating the sound transmission through a fine grid by means of the added mass of its elements is well known to the specialist, although it was never published by its author. A new proof of Maliuzhints' formula is given below, with the consent of its original author.

First a few preliminary remarks will be made. It is assumed that the fluid is ideal and compressible. The pressure p is a function of the density ρ only. The velocities of the fluid particles are so small that their squares may be neglected in comparison with the first powers of them. The variations in density and pressure are also small.

The flow (a plane sound wave) possesses a velocity potential Φ . If this potential can be represented in the form $e^{i\omega t}f(x, y)$, then it satisfies the wave equation (cf., for example, [1])

$$\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}=-k^2\Phi \qquad \left(k=\frac{\omega}{c}\right) \tag{1}$$

where c is the velocity of sound. It is known that the intensity of sound is measured by the flow of energy carried by the progressive waves through a unit of area. We shall consider a plane sound wave with the potential

$$\Phi = A\cos(\omega t - kx) + B\sin(\omega t - kx)$$

which satisfies (1). It is not difficult to calculate the intensity of sound for such a flow:

$$\frac{\omega}{2\pi}\int\limits_0^{2\pi/\omega} p\frac{\partial\Phi}{\partial x}\,dt = \frac{\rho\omega k}{2}(A^2+B^2) \tag{2}$$

The behavior of the velocity potential $\Phi(x,y)$ at large distances from the grid may be investigated with the help of Rayleigh's method [2]. The investigation of the flow of fluid in front of the grid and behind it may be carried out by similar procedures. Thus we shall limit our consideration to the flow behind the grid. Let us take a plane sound wave having a potential of the form $A\cos(\omega t - kx + \theta_0)$, where A and θ_0 are arbitrary constants. We place in our flow a grid with axis parallel to the y-axis. Let the y-axis be very close to the right-hand side of the grid and let the screen have period $l = 2\pi/p$. We shall consider the flow in the half-plane $x > 0$. Since its potential is clearly proportional to the velocity of the initial flow for $x = 0$, then we shall assume that for $x = 0$ the following relation is valid for the potential Φ

$$\Phi_{x=0} = a_0 + a_1\cos py + b_1\sin py + \ldots + a_n\cos npy + b_n\sin npy \ldots$$

where a_n and b_n are linear combinations of $\cos \omega t$ and $\sin \omega t$. For $x>0$ we seek a potential of the form

$$\Phi = a_0 A_0 (x) + \ldots + a_n A_n (x) \cos npy + b_n B_n (x) \sin npy + \ldots \qquad (3)$$

From Equation (1) it follows that $d^2 A_0/dx^2 = -k^2 A_0$. Consequently, A_0 is a linear combination of $\cos kx$ and $\sin kx$. The corresponding term in (3) gives the potential of the transmitted wave. Further, in view of Equation (1), A_n (and also B_n) satisfy Equation

$$d^2 A_n/dx^2 = (n^2p^2 - k^2) A_n$$

If the structure of the grid is fine, i.e. $p > k$, then

$$A_n = C_1 e^{-x\sqrt{n^2p^2-k^2}} + C_2 e^{x\sqrt{n^2p^2-k^2}}$$

where C_1 and C_2 are constants. It is physically clear that as $x \to \infty$ the amplitude must remain finite, and hence $C_2 = 0$. Therefore the velocity field behind the grid consists of a transmitted wave and a flow with a potential decaying at least as $\exp(-x\sqrt{p^2-k^2})$ as $x \to \infty$.

If the structure of the grid is sufficiently fine, then we assert that it is not difficult to generalize to the case where the axis of the grid is not perpendicular to the direction of the sound wave.

An analysis similar to that just given shows that in this more general case the flow behind the grid will consist of a transmitted wave and a flow whose potential decays exponentially as $x \to \infty$, as long as the angle between the axis LL of the grid and the x-axis (see Fig.1) is not small.

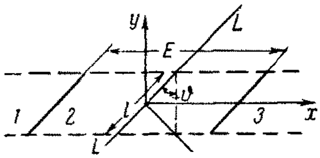

Fig. 1

We now proceed to the derivation of the formula of Maliuzhints for sound transmission through a fine grid. If we place in the path of a sound wave moving along the x-axis a grid whose period and thickness are small in comparison with the wavelength $\lambda = 2\pi/k$, then part of the wave will pass through the grid, part will be reflected, and furthermore, there will be a flow which is noticeable only in the immediate neighborhood of the grid.

We fix the origin of coordinates within one of the elements of the grid. Let the potential of the incident and reflected waves in front of the grid be

$$\Phi^{(1)} = A \cos (\omega t - kx) + B \sin (\omega t - kx + \beta) + C \sin (\omega t + kx + \gamma)$$

Let $\Phi^{(3)} = A \cos (\omega t - kx)$ be the potential of the transmitted wave. The parameters A, B, C, β and γ are related so that the flow to the left of the grid joins continuously with the flow to the right. After this the coefficient of sound transmission of the grid is found quite readily. On account of the smallness of the dimensions of the grid compared to the wavelength, the velocity of the flow impinging on the grid is equal to

$$U \approx [\partial\Phi / \partial x]_{x=0} = kA \sin \omega t - kB \cos (\omega t + \beta) + kC \cos (\omega t + \gamma)$$

The velocity immediately behind the screen is equal to

$$U^{(3)} \approx [\partial\Phi^{(3)} / \partial x]_{x=0} = kA \sin \omega t$$

Because of the continuity of the flow and the incompressibility of the liquid near the grid, it must be assumed that $U^{(1)} = U^{(3)}$ whence $B = C$ and $\beta = \gamma$, and the velocity of the flow impinging on the grid equals

$$U = k\,A \sin \omega t \qquad (4)$$

We shall now consider the picture of the flow in more detail. We choose the element of the grid containing the origin of the coordinate system and the band of flow from $x = -\infty$ to $x = \infty$ which corresponds to this element.

There is no need to specify the form of this band more precisely; it will suffice to assume that its upper and lower boundaries are displaced from each other by the spacing of the grid. We shall divide this band into three regions (see Fig. 1).

R e g i o n 1 extends from $x = -\infty$ to $x = 0$. The potential in this region is

$$\Phi = \Phi^{(1)} = A\cos(\omega t - kx) + B[\sin(\omega t - kx + \beta) + \sin(\omega t + kx + \beta)]$$

R e g i o n 2 is in the neighborhood of $x = 0$; its dimensions in all directions are small in comparison with the wavelength (i.e. $k\sqrt{x^2 + y^2}$ is small in this region), but ε – the length of region 2, although infinitely small compared to the wave length, is infinitely large in comparison with the period of the grid. The flow in region 2 may be treated as the flow of an incompressible fluid ([1], Chapter X, Section 290, 305), impinging on the grid with velocity U.

The flow in region 2 consists of the flow caused by a grid moving with velocity $-U$ (potential φ) and a uniform flow with potential U_x

$$\Phi = \Phi^{(2)} = \varphi + Ux$$

R e g i o n 3 extends from $x = \infty$ to $x = 0$. The potential in it is

$$\Phi = \Phi^{(3)} = A\cos(\omega t - kx)$$

We shall noe calculate the difference $\delta\Phi$ in potential between the right and left boundaries of region 2 by two different methods. On the one hand, neglecting in each of the terms $\Phi^{(1)}$ and $\Phi^{(3)}$ the small quantities $|kx|$ of higher order, we obtain

$$\delta\Phi = k\varepsilon A\sin\omega t - 2B\sin(\omega t + \beta) \tag{5}$$

On the other hand, the same difference in potential is equal to

$$\delta\Phi = U\varepsilon + \delta\varphi \tag{6}$$

where $\delta\varphi$ is the potential difference between the right and left boundaries of region 2. As a consequence of the stated assumptions regarding the structure of the grid and the dimensions of region 2 we have

$$\delta\varphi \approx \varphi_{(x\to\infty)} - \varphi_{(x\to-\infty)} = \varphi_\infty - \varphi_{-\infty} \tag{7}$$

From (4) to (7) follows the approximate equality

$$-2B\sin(\omega t + \beta) = \varphi_\infty - \varphi_{-\infty} \tag{8}$$

In using the theorem of change in momentum, we can calculate $\varphi_\infty - \varphi_{-\infty}$ by means of the added mass and area of an element of the grid. According to Sedov [3]

$$\lambda_{11}U + i\lambda_{12}U = -\rho SU + i\rho\int z\,dw, \qquad w = \varphi + i\psi, \qquad z = x + iy \tag{9}$$

where S is the area of an element of the grid, λ_{11} is the added mass in the x-direction, and w is the complex potential of the flow for a moving grid. The integral is carried out along any closed contour which encircles the element of the grid once. It may be shown that at infinity to the left and right

$$z\left(\frac{dw}{dz}\right)_{x=\pm\infty} = 0 \tag{10}$$

Therefore, extending the contour of integration up to the boundaries of region 2, i.e. from $x = -\infty$ to $x = \infty$ as compared with the period and thickness of the grid, and using the fact that the complex velocities dw/dz are equal at those points on the upper and lower boundaries which differ by the period $l(\sin\vartheta + i\cos\vartheta)$ we obtain

$$\oint z\,dw = -l(\sin\vartheta + i\cos\vartheta)(w_\infty - w_{-\infty}) = -l(\sin\vartheta + i\cos\vartheta)(\varphi_\infty - \varphi_{-\infty})$$

Hence, using (9), we obtain

$$\varphi_\infty - \varphi_{-\infty} = \frac{kA \sin\omega t\,(\lambda_{11} + \rho S)}{\rho l \cos\vartheta} \qquad (11)$$

Substituting $\varphi_\infty - \varphi_{-\infty}$ from (11) into (8), we find that

$$-2B\sin(\omega t + \beta) = \frac{kA \sin\omega t\,(\lambda_{11} + \rho S)}{\rho l \cos\vartheta}$$

whence

$$\beta = 0, \quad B = -\frac{kA\,(\lambda_{11} + \rho S)}{2\rho l \cos\vartheta}$$

Then using Equation (2), we find that the average energy E_1 carried by the incoming wave, and the energies E_2 and E_3 of the transmitted and reflected waves are, respectively,

$$E_1 = \frac{\rho\omega k}{2} A^2 \left[1 + \frac{k^2(\lambda_{11} + \rho S)^2}{4\rho^2 l^2 \cos^2\vartheta}\right], \quad E_2 = \frac{\rho\omega k}{2} A^2, \quad E_3 = \frac{\rho\omega k}{2} A^2 \frac{k^2(\lambda_{11} + \rho S)^2}{4\rho^2 l^2 \cos^2\vartheta}$$

Hence we have for the coefficients of sound transmission α and reflection r

$$\frac{1}{\alpha} = 1 + \left[\frac{\omega(\lambda_{11} + \rho S)}{2\rho C l \cos\vartheta}\right]^2, \qquad \frac{1}{r} = 1 + \left[\frac{2\rho C l \cos\vartheta}{\omega(\lambda_{11} + \rho S)}\right]^2$$

Thus the knowledge of the added mass of the grid enables one to calculate the coefficients of sound transmission and reflection of the grid, that is, to settle the question of how much of the sound is transmitted and how much reflected.

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